

A set of integers is defined by

1. 2 is in the set
2. If  $n^2$  is in the set then  $n$  is also in the set
3. If  $n$  is in the set then  $(n+5)^2$  is also in the set

What integers are not in the set?

My solution

From rules 2 and 3 we will add an extra rule

4. If  $n$  is in the set then  $n+5$  is also in the set

Rule 4 means that we can write the integers down in 5 columns starting with 1, 2, 3, 4, 5. And whenever we identify an integer which is in the set then all integers in the same column below it must also be part of the set.

My working, actually the result of many iterations is shown below.

This number	Generates	Using rule
2	5, 7, 12, ... (All column 2 is in set)	4
2	49	3
49	$54^2$	3
$54^2$	$56^2$ (since both end in 6)	4
$56^2$	56	2
56	121	4
121	11	2
11	36	4
36	6	2
6	11, 16, 21, ... (note especially 16) (all column 1, <b>except 1</b> , is in set)	4
16	4	2
4	9, 14, 19, ... (note especially 9) (all column 4 is in set)	4
9	3	2
3	8, 13, 18, ... (all column 3 is in set)	4

This includes all integers except 1 and all multiples of 5. I don't believe you can jump into column 5 from any of the other columns using just the 4 rules, above. Also don't believe you can get 1.

My answer: the only integers NOT in the set are 1 and all multiples of 5.