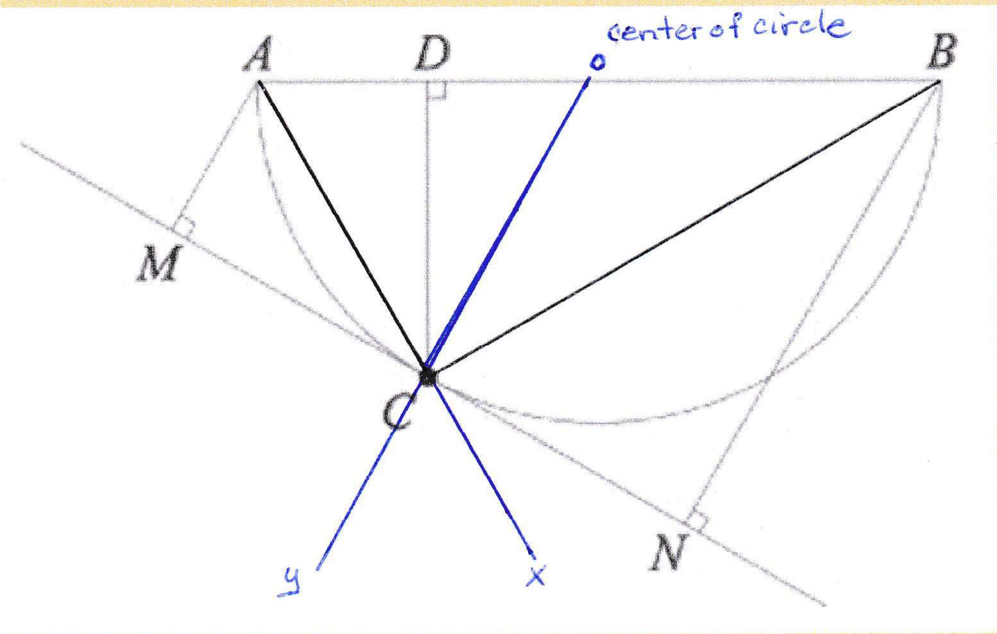
GIN Puzzle Bent 2026 | puz 3

In the accompanying drawing below, C is a point on the semicircle with AB as diameter. MN is tangent to the semicircle at C. AM and BN are perpendicular to MN, and CD is perpendicular to AB. Find an expression for the length of the segment $\overline{CD}$ as a function of the lengths of $\overline{AM}$ and $\overline{BN}$.



① Angle BCA $= 90^\circ$ Thales's theorem

② Triangles ACM & ACD are congruent ⑤

③ Triangles BCN & BCD are congruent (similar to ⑤)

From 2

$AC^2 = CD^2 + AD^2$; $AD = AM \Rightarrow AC^2 = CD^2 + AM^2$

From 3

$BC^2 = BN^2 + CN^2$; $CD = CN \Rightarrow BC^2 = BN^2 + CD^2$

$AC^2 + BC^2 = CD^2 + AM^2 + BN^2 + CD^2$ ④

From 1

$AB^2 = BC^2 + AC^2$; $AB = AD + BD$; $AD = AM$ & $BD = BN$

$(AM+BN)^2 = BC^2 + AC^2$

④ $(AM+BN)^2 = CD^2 + AM^2 + BN^2 + CD^2$

$\cancel{AM^2} + 2AM\cdot BN + \cancel{BN^2} = 2CD^2 + \cancel{AM^2} + \cancel{BN^2}$

$CD = \sqrt{AM \cdot BN}$

QED ☺

⑤

$\triangle ACO$ is isosceles $\therefore \measuredangle OAC = \measuredangle OCA$

AC intersects AM & OC which are $\parallel$ $\therefore \measuredangle MAC = \measuredangle XCY = \measuredangle OCA = \measuredangle OAC$

$\measuredangle MAC = \measuredangle OAC$

2 Right $\triangle$s with same hypotenuse & $=$ $\measuredangle$s are congruent

$\therefore \triangle ACM \cong \triangle ACD$