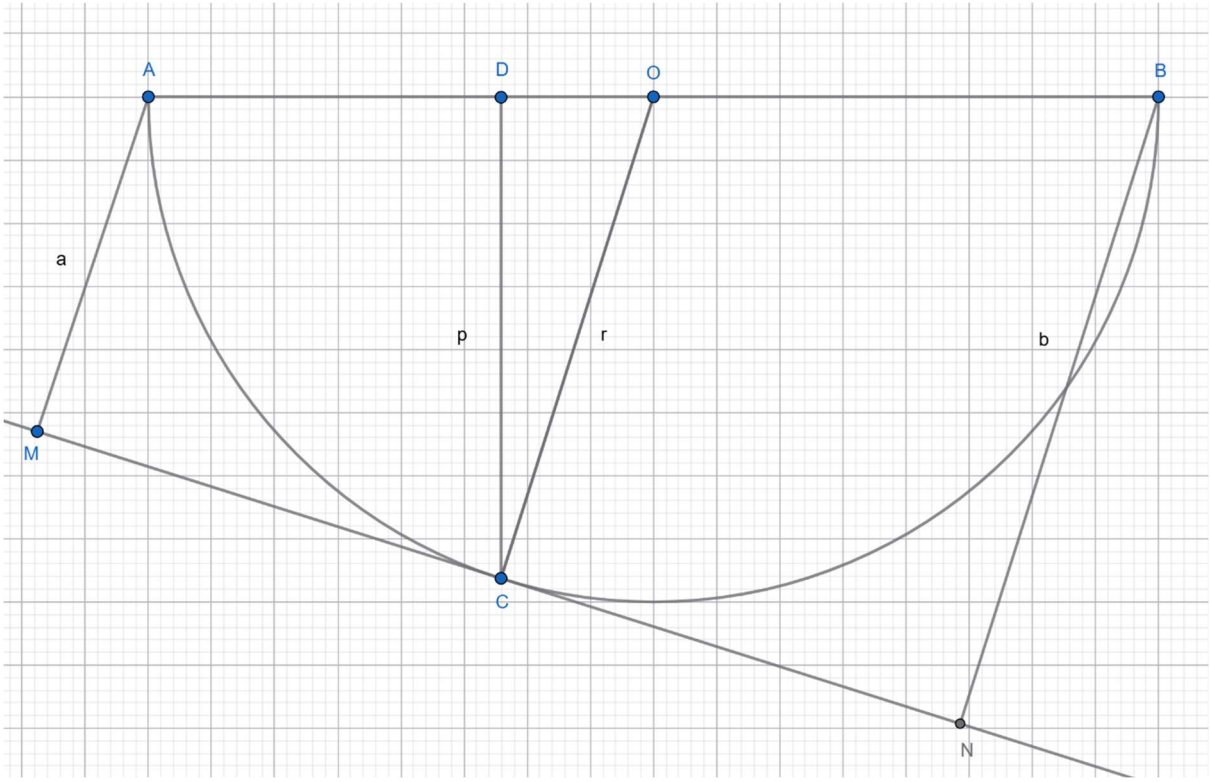
Nathan's hemispherical puzzle:



Let a = AM, r = OC, P = Bn

By inspection

$$r = (a + b)/2$$

Now

$$p = r\cos\theta$$

and

$$sin\theta = \frac{b - a}{2\,r}$$

giving

$$cos\theta = \sqrt{\left(1 - \left(\frac{b - a}{2r}\right)^2\right)}$$

Substitute for 2 r

$$cos\theta = \frac{\sqrt{(b+a)^2-(b-a)^2}}{a+b} = \sqrt{4ab}/(a + b)$$

So $p = r\,cos\theta = \sqrt{(a\,b)}$